ELSEVIER Journal of Hazardous Materials 37 (1994) 459–473

JOURNAL OF HAZARDOUS MATERIALS

A model for enhanced aeration of streams by motor vessels with application to the Seine river

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(Received 14 January 1993; accepted in revised form 24 September 1993)

Abstract

The transport of molecular oxygen, a natural chemical, into water from air is of critical importance in maintaining water quality of bayous, canals, waterways, rivers, harbors, estuaries, etc. Research on this classical environmental engineering intermedia transport process has received considerable attention for decades, however its ongoing significance demands continued research. The Seine river in France is a typical example of a river that serves multiple purposes and a detailed analysis of all the oxygen transport mechanisms is needed. Following a brief review of the known mechanisms of O_2 transport a combined momentum balance plus propeller surface aerator model is proposed to account for reaeration by motor vessels moving on waterbodies. The potential importance of this mechanism is put into context through a brief case study of the Seine where flow and dam hydraulics induced reaeration, along with wind-enhanced reaereation, are quantified. The primary objective of this paper is to develop a model for estimating the aeration contributed to streams and waterbodies by vessels moving on the surface. First the general subject of the oxygen balance and stream reaeration is introduced. This is followed by a brief review of the two natural aeration mechanisms, flow and wind, that are active in all streams. The environmental situation at Porcheville, France on the Seine river 75 km downstream of the center of Paris, is parameterized for developing numerical estimates of the aeration process for each mechanism. After the theoretical model for vessel aeration is developed a case study of aeration on the Seine is used to illustrate the potential significance of vessel aeration in the context of other mechanisms including dams. The paper concludes with statements on the state-of-knowledge and the scientific/engineering needs on the subject.

1. Introduction

Molecular oxygen input through the air/water (A/W) interface is a vital process for the health of a river ecosystem. The Seine is a typical example of a river expected to

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SSDI 0304-3894(93)E0101-7

serve many uses. These include water supplies, waste treatment, navigation, power generation and recreation. To a degree, all these uses affect the natural oxygenation capacity, normally termed aeration or reaeration capacity, of the river. All things considered, oxygen is possibly the most important chemical constituent of rivers other than H_2O.

The oxygen balance is one of several water quality constituent balances needed to quantify chemical fate in rivers. The following equation from Even and Poulin [1] is typical of such transient models for a fixed stream volume:

$$\frac{dc}{dt} = \alpha_0 - \beta - b - \mathrm{SOD} + k_2(C^* - C) \tag{1}$$

Here the rate of change of oxygen concentration, C, is determined by five terms that account for: O_2 production by photosynthesis, consumption by algal respiration, consumption by heterotrophic and nitrifying bacteria and sediment oxygen demand (SOD). The fifth term accounts for reaeration through the A/W interface where the coefficient k_2 has dimensions of t^{-1}, typically d^{-1}. The concentration difference is the displacement from saturation, C^* being the thermodynamic solubility limit of O_2 in water at the local conditions of temperature, pressure and presence of other substances such as NaCl. The k_2 coefficient in Eq. (1) contains a river capacity factor; it includes the depth, h. A more general coefficient for interphase chemical transport is defined by the rate expression for the flux of O_2 per unit stream surface area,

$$n_0 = k'(C^* - C) \tag{2}$$

(See ref. [2, pp. 78–81].) This coefficient, k_2' which has dimension of velocity is related to the river reaeration coefficient in Eq. (1) through $k_2' = k_2 h$. It will be used throughout this report.

It is common in oxygen balance models such as Eq. (1) to treat the reaeration coefficient as a single constant. In most applications it is estimated and adjusted so it correspond to the available oxygen data for the stream. In this way it serves well for most uses such as waste load allocation and other normal and/or average conditions to predict dissolved oxygen concentration. However, this single, lumped parameter, coefficient approach will not suffice as the expectations of such models increase. As demands on the stream resources increase so will the demands on the predictive capabilities of water quality models. These demands include primarily transient phenomena such as storm runoff inputs, wastewater plant upset condition, extremes in natural environmental processes that include flow, wind, temperature, sunlight, etc.

With these demands in mind the oxygen transport coefficient, k_2' will be dissected into its component parts in the following sections. In turn, the contributions of flow, wind, dams and motor vessel traffic will be addressed. This approach of analyzing the mechanism of oxygen input into streams provides for better modeling and it also suggests directions of possible manipulations of river processes that may lead to improved aeration.

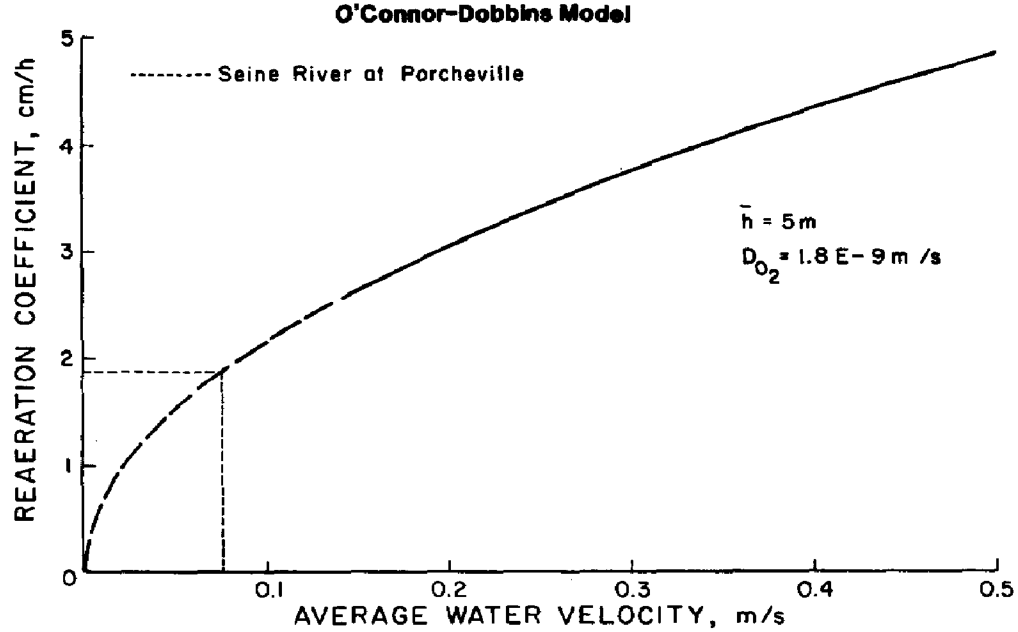

Fig. 1. Reaeration of water by flow.

2. Flow aeration

Considerable research over the last seven decades on the natural aeration process due to flow has led to the development of field-tested algorithms for predicting the reaeration coefficient, k_2'. Three general categories have been devised for rivers: deep and slow, large and swift and shallow and swift [3]. At Porcheville the Seine falls within the deep and slow category and the O'Connor and Dobbins model applies:

$$k_2' = (DV/h)^{1/2} \tag{3}$$

Here D is the molecular diffusivity of O_2 in water at its temperature (m^2/s), V is the average water velocity (m/s), h is the average water depth (m) and the coefficient is in m/s. The suggested range of applicability of the O'Connor–Dobbins algorithm is V of 0.15 to 0.5 m/s and h of 0.30 to 9.1 m. The reaeration coefficient is presented graphically in Fig. 1 for 20 °C with $D = 1.810^{-9}$ m^2/s.

3. Wind aeration of quiescent waters

Wind moving over a waterbody induces drag upon surface. The transferred interfacial shear stress results in a surface water velocity of approximately 3.5% that of air. The aeration of water by wind has been studied in detail in both the laboratory and in the field [2, 3]. Some of the most recent work has been performed by Lunney et al. [5]

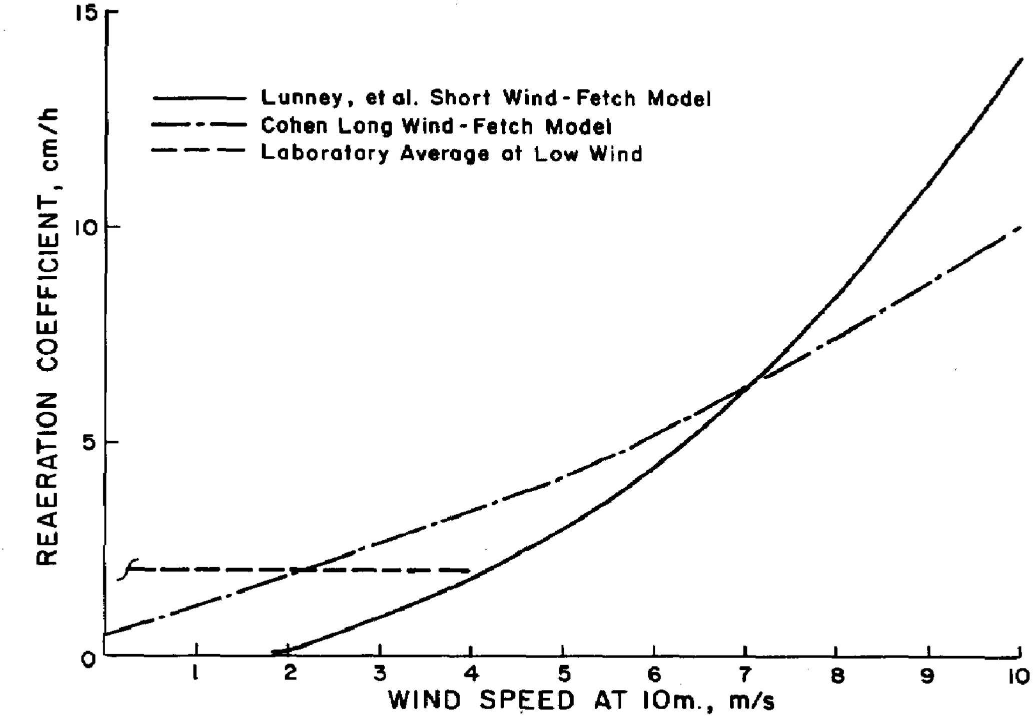

Fig. 2. Reaeration of quiescent water by wind.

and Cohen [4]. Both works report coefficients that are a strong function of wind speed. That of Lunney et al. is

$$k_2' = 118 D^{2/3} V_A^{2.23} \tag{4}$$

for $V_A > 4.5$ m/s and $k_2' = 2$ cm/h for $V_A < 4.5$ m/s. Here k_2' is in cm/h, D is molecular diffusivity of chemical in cm^2/s and v is the wind speed at 10 m elevation in m/s. Fig. 2 is a graphical representation of Eq. (4) for $D(O_2) = 1.810^{-5}$ cm^2/s. This result is more applicable for waterbodies with a short wind-fetch. The coefficient does not go through the origin for periods of no wind. Other factors, not all of them known but including capillary waves and gentle residual fluid motions plus thermal gradients create a low level of turbulence and maintain a finite coefficient. Most data sets display a 1 to 3 cm/h coefficient in the $V_A < 5$ m/s range [2]. Cohen's model was developed for marine waters and a long wind-fetch. The equation is

$$k_2' = 0.443 + 7.33\, V_*^{1.015} \tag{5}$$

where the coefficient is in cm/h. Here V_*, in cm/s, is the friction velocity related to the wind velocity by

$$V_* = 3.5\sqrt{C_D V_A} \tag{6}$$

where $C_D = 8.510^{-4}$ for $V_A \le 5$ m/s, $C_D = 8.510^{-4} + 1.110^{-4}$ $(V_A - 5.0)$ for $5 < V_A \le 20$ and $C_D = 2510^{-4}$ for $V_A > 20$ m/s. Fig. 2 contains a graphical representation of Eq. (5). The correlation has a non-zero intercept of 0.443 cm/h, however Cohen's laboratory data shows k'_2 to be approximately 2 cm/h for $V_A \leq 3.5$ m/s. The data from laboratory experiments for low wind speed is also represented in Fig. 2.

4. Motor vessel aeration of quiescent waters

To date, no information has been found on this subject. What follows is a model for motor vessel aeration of streams. It consists of two parts, one being a momentum balance model for the hull moving through the water; the other is a mechanical surface aerator model for the contribution to aeration provided by the propulsion device.

4.1. *Momentum balance model*

Motorized and sailing vessels moving upon the surface of a waterbody create a localized hydraulic head due to the volume of water displaced. As a boat moves through the water, it digs a hole in the water, creating a bow wave and a stern wave. The distance between these waves is basically the waterline length of the boat. Although these waves move diagonally relative to the centerline of the boat, there is a stern wave that runs perpendicular to the boat's course and also moves at the same speed as the boat. This is the wave sailors use when they want to be 'towed' by faster boat in competitive sailing. The action is similar to the drafting technique used by racecar drivers and bicycle racers. This localized head imparts artificial water flow in addition to any existing flow. An observer stationed on the bank of a small stream cannot avoid noticing the induced water velocities in advance of, during and subsequent to the passage of a vessel. Fig. 3 illustrates these three aspects of the flow field in a narrow channel. A bow wave is formed forcing water away from the advancing vessel (1). In a restricted channel this artificial mound of water seeks relief by flowing rapidly past the vessel in the opposite direction (2). In large channels this flow is less apparent. Once the vessel passes returning flow fills the void left by the advancing

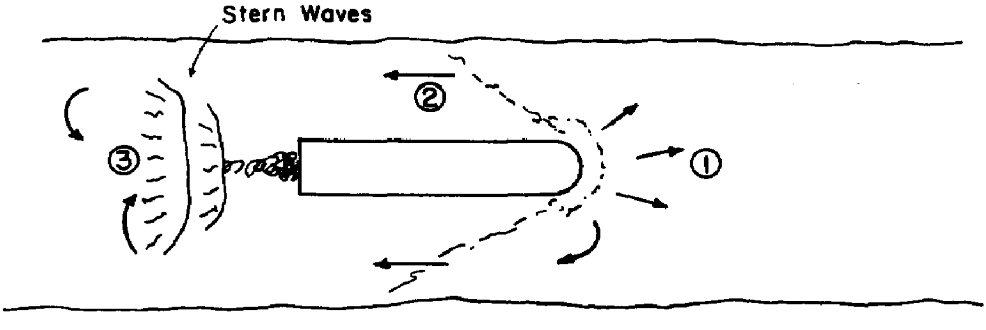

Fig. 3. Motor vessel creating artificial flow in channel.

vessel (3). Obviously the flow processes around and away from a moving vessel is very complex as would be a complete fluid dynamic treatment if it were possible. The following theoretical approach is a balance between fluid dynamic rigor and engineering practicality. Basically the approach is to use the energy balance for the energy delivered by the vessel's hull in order to obtain the shear stress, both form and friction, imparted to the water column. This stress is then assumed to be dissipated by the stream through an equivalent hydraulic head and bottom friction yielding an effective water velocity, V_e. Although this velocity cannot be measured in the stream it is a characteristic parameter reflective of the process in which energy is imparted by the vessel and then dissipated in the stream. In other words, V_e can be interpreted as an auxiliary flow velocity in the waterbody due to the passage of the vessel. Assuming that the net effect of this auxiliary flow is analogous to the natural hydraulics in streams then V_e can be used with any of several stream reaeration alogrithms to obtain a coefficient, k_2', attributed to the vessel.

Manning's formula [8] is a convenient and common means of obtaining the velocity in the channel from available river characteristics. In SI units the equation is

$$V = r^{2/3} S^{1/2} / n \tag{7}$$

where V is mean velocity m/s, r is hydraulic radius m, S is the hydraulic gradient and n is the coefficient of roughness. Tabulated values of n are available in hydraulic handbooks. The river bottom shear stress induced by the flow gradient S can be obtained by

$$\tau = \rho S h g \tag{8}$$

where τ is the stress in N/m^2, ρ is the density of water, kg/m^3, h is the water depth, m, and g is gravitational acceleration m/s^2. Ideal, shaped objects moving through quiescent water will be used to model the momentum imparted by the motor vessel. A half sphere for the bow and another for the stern plus flat plates for the bottom and sides form an idealized vessel, see Fig. 4. The dimensions: draft, h_v (m); width, w_v (m) and length, l_v (m) characterize the vessel where $\sqrt{4h_v w_v/\pi}$ is reasonable for the equivalent sphere diameter, d_v (m), since the dimensions h_v and w_v are similar.

The friction force for both form and skin is commonly quantified by

$$F = A[\rho V_v^2] f \tag{9}$$

where F is the force (N) on the object, A is its area (m^2), V_v is the velocity of fluid past (m/s) and f is the friction factor. Bird et al. [6] give the following for each shape:

$$F_s = \frac{1}{4}\pi d_v^2 \left(\frac{1}{2}\rho V_v^2\right) f \tag{10}$$

where $f = 0.44$, Newton's law region for turbulent flow past a sphere. For a plate

$$F_p = (2h_v + w_v) l_v (\rho V_v^2) f \tag{11}$$

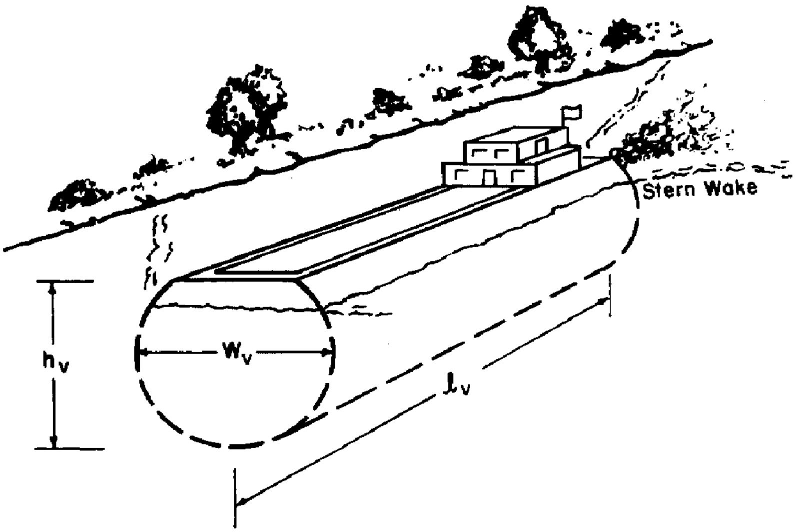

Fig. 4. Idealized geometric model for motorized vessel.

where $f = 0.074/(l_v V_v/\nu)^{1/5}$, ν is kinematic viscosity of water; this expression is the drag on a flat plate in turbulent flow. The total drag force on the ideal vessel is the sum of that in Eqs. (10) and (11). The effective force is therefore

$$F_t = F_s + F_p \tag{12}$$

for a single vessel.

A series of vessels moving past a fixed point, at a rate N per unit time (s^{-1}) deliver their combined energy to the river. As argued previously this energy will be dissipated in bottom shear stresses. For vessels of speed V_v the number per unit area of river surface area is $N/V_v W$ so that the effective stress can be estimated by

$$\tau_v = F_t N/V_v W \tag{13}$$

where τ_v is the shear stress in N/m^2 and W is the width of the river, m. Eqs. (10)–(13) are combined to yield τ_v.

Through Eqs. (7) and (8) it is possible to work backwards, knowing the bottom shear stress equivalents, to estimate an effective water velocity, V_e, due to the vessels. Combined, the resulting expression for V_e becomes

$$V_e = h^{2/3}(\tau_v/h\rho g)^{1/2}/n \tag{14}$$

where $r \approx h$, the depth. By the use of the O'Connor–Dobbins or a similar reaeration algorithm this effective water velocity, along with the water depth, yields the coefficient for the motor vessel. This is the procedure used for estimating the so-called momentum balance coefficient.

Sample calculations were performed that reflect the Seine environment at Porcheville Yacht Club in September, 1991. Here the depth is 5 m, width 1000 m and $n = 0.035$ is assumed. Small vessels were assumed to have width and draft of 4 m with

Table 1
Reaeration – momentum balance model prediction

N (vessels/h)	Small vessels		Large vessels	
	Power (W)	Coefficient (cm/h)	Power (W)	Coefficient (cm/h)
1	14314[a]	0.984	51122[b]	1.35
3	42942	1.29	102244	1.77
5	71571	1.46	255610	2.01
10	143141	1.74	511219	2.40
20	286282	2.07	1022000	2.85

[a] 19.2 hp.
[b] 68.6 hp.

length of 30 m. The vessels were assumed to have quiescent water speed of 5 knots (1 knot = 0.514 m/s). Water viscosity was 0.01 cm^2/s and oxygen diffusivity 1.810^{-9} m^2/s are for 20 °C. Large vessels were assumed to be nine small vessels creating a 'tow' 3 vessels wide and 3 vessels deep for a unit that was 12 m in width, 4 m in draft and 90 m length. Table 1 contains the numerical results of the calculation for assumed values of vessel traffic rate. The reaereation coefficients are also presented graphically in Fig. 5. The significance of these coefficients will be discussed later.

The power input by the propellers is P. A portion of this, P_d, goes into thrust to move the vessel and overcome the drag of water on the hull. It can be computed by $P_d = \rho_v V_v^2 W/N$. The P is larger than P_d, the excess power, P_m, going into mixing the water. Clearly $P = P_d + P_m$. Since the portion of shaft power converted into thrust is unknown, a value of 50% will be assumed. This means that $P_d/P = 0.5$ or $P_m = P_d$. For the calculated power values, P, appearing in Table 1 the above relationships were used. This is the power delivered to the water and is approximately the power delivered to the shaft by the engine.

4.2. *Surface aerator model*

Motorized impeller devices placed upon the surface of a waterbody create high outward surface water velocities to enhance oxygen transport. Unlike the propellers on motorized watercraft impellers are typically installed very near the surface with the plane of rotation parallel to the surface. Propellers are usually installed much deeper to develop thrust so that the energy expended creating turbulence is at depth rather than near the surface. The net effect being that boat propellers may be less efficient O_2 transfer devices; nevertheless, a surface aerator model will be developed for propeller driven watercraft.

A standard correlation for surface aerators oxygen transfer rate is presented by Thibodeaux [2] (Eq. 4.1 C-1, p. 64). Converted to an O_2 reaeration coefficient form,

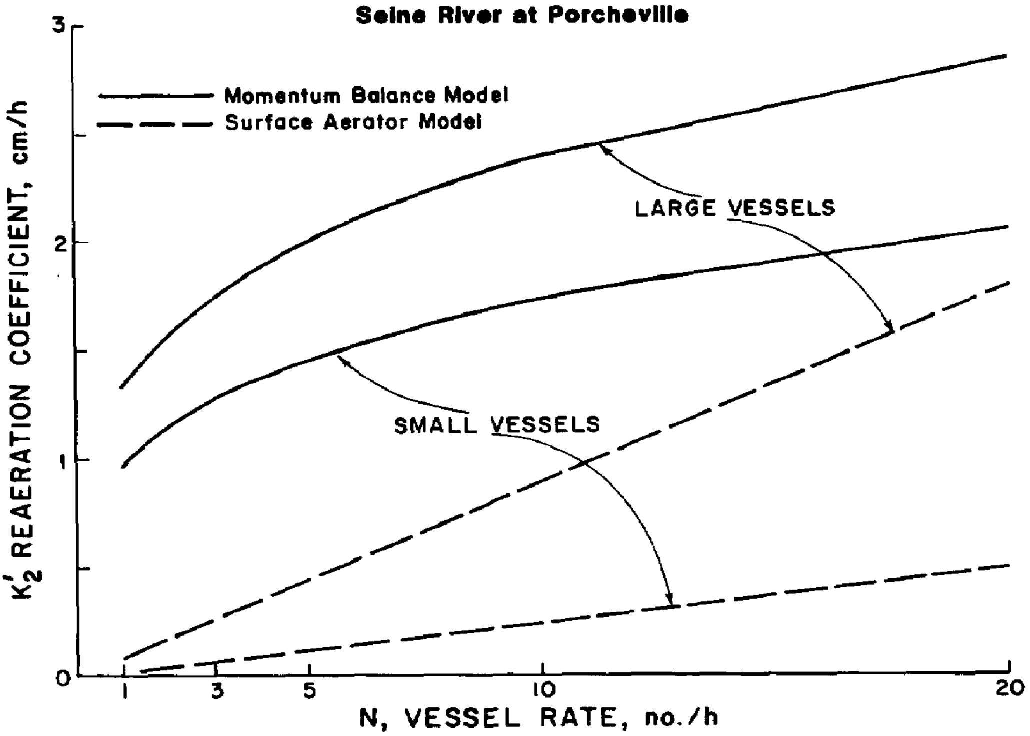

Fig. 5. Reaeration of quiescent water by motor vessels.

the equation becomes:

$$k'_2 = 10.9\,NREP_m\alpha(1.024^{T-20°C})/V_vW \qquad (15)$$

in which the following units have been used: R is the rated oxygen deliver rate (1.2 to 4.1 g/W h); E is the motor efficiency (0.65 to 0.90); P_m is the nameplate power (W); α is a clean/dirty water correction factor (0.8 to 1.0); and T is water temperature (°C). The coefficient is in cm/h. V_v in m/h, W in m and N in h^{-1}.

Sample calculations were performed with the same assumptions and data used previously for the momentum balance model. The product EP_m is taken half of the power values in Table 1.

In addition $R = 3.0$, $T = 20$ °C and $\alpha = 1.0$ was assumed. The calculations for the Porcheville site appear in Table 2 and again graphically in Fig. 5. The significance of these coefficients is discussed below.

Clearly it can be argued that the aeration performed by motor vessels in quiescent waters cannot be adequately quantified by either model. However, these vessels appear to have features of both. Due to bulk water volume displacement of the hull combined with an impeller type aerator in the stern, the most appropriate model should combine these features. In this case the effective aeration coefficient is simply

Table 2
Reaeration – surface aerator model prediction

N (vessels/h)	Coefficient (cm/h) Small vessels[a]	Large vessels
1	0.025	0.090
3	0.076	0.271
5	0.126	0.452
10	0.253	0.903
20	0.506	1.81

[a] See Table 1 for vessel power.

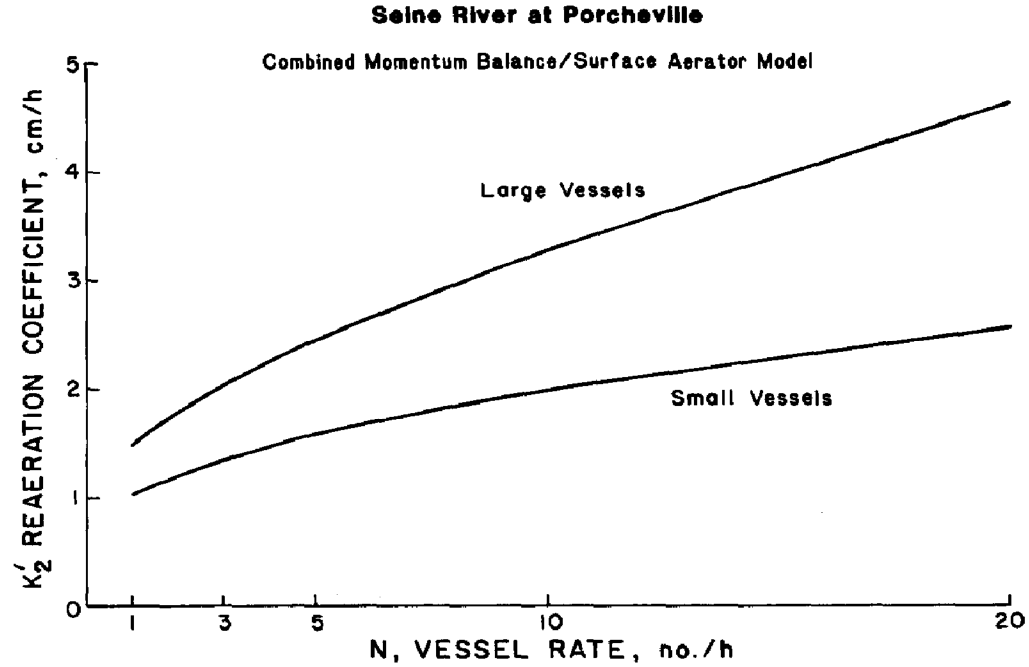

Fig. 6. Reaeration of quiescent water by motor vessels.

the sum of the momentum balance and surface aerator contributions. Fig. 6 contains this combined model for both small and large vessels on the Seine.

5. Discussion

In the previous sections three modes of oxygen transport through the A/W interface of rivers were considered. These were aeration by: (1) water flowing naturally down its hydraulic gradient; (2) by wind moving over and interacting with the surface; and (3) by motor vessels moving on the surface. The theory behind each point was outlined for (1) and (2) while detailed derivations were presented for (3) because of the apparent lack of information on this subject.

Conditions observed on the Seine at Porcheville on September 18, 1991 were used in numerical calculations to assess the contribution of each mode of oxygen transport to the overall aeration capacity of the river. Fig. 1 shows the flow hydraulics coefficient of O'Connor–Dobbins model. At this location the water temperature was 19 °C, velocity on the surface 7.7 cm/s and depth 5 m. The model predicted coefficient is 1.9 cm/h (see Fig. 1). The input velocity is less than the recommended lower value so that the coefficient is better described as being less than about 2.5 cm/h. It is reasonable to expect a low value of this coefficient at this place and time of year. Overall aeration was low; dissolved oxygen was measured at 1.5 mg/l and the water surface appeared more like the surface of a lake than a flowing stream!

Winds are light in Paris in the summertime. Data in July, August and September for 1989, 1990 and 1991 reveal that monthly averages are 2.5 to 3.1 m/s with few 10 min, average, excursions over 8 m/s (METEO FRANCE, Station C.I.D.M. Montsouris). On September 18 the wind averaged 1.6 m/s with one 10 min excursion of 5 m/s velocity. This was also the situation at Porcheville. The wind was very light. Occasionally small patches of ripples appeared on the water surface. These were caused by light wind gusts events but most of the time between 1000 and 1800 h the surface was smooth. The graph in Fig. 2 suggests that the wind generated aeration coefficient was no greater than 2 cm/h. If this is correct the flow hydraulics coefficient and the wind-induced coefficient are approximately equal. At a wind speed of 8 m/s the coefficient increases to 8.5 cm/h, however, many 10 min excursions must occur in a 24 h period to product any appreciable effect on the overall coefficient.

Two complimentary, motor driven watercraft aeration models were developed and the behavior of these two theoretical models, appear in Fig. 5 and are quite different. With increasing vessel passage rate (i.e, N) the coefficient for the momentum model rises very rapidly and plateaus to a near constant values for large and small vessels. It levels off at 2 to 3 cm/h for $N = 20$/h. The surface aerator model behavior is linear with N and the numerical values are nearly half the momentum model values. Values of the coefficient increase rapidly with N for the case of large vessels reaching nearly 2 cm/h for $N = 20$/h. Here it is only 0.5 cm/h for small vessels. It was argued above that the more realistic model combines the two and this behavior is depicted in Fig. 6. Clearly the momentum balance aeration process dominates the shape of the curves.

It was estimated that the traffic rate on September 18 at Porcheville was about 10 vessels/h. There was a constant movement of vessels up and down the river and at any given time one was usually in sight, either coming or going. Many more small vessels passed than large; roughly one large for every 5 to 10 small ones. If this was in fact the situation the vessel passage generated coefficient was above 2.0 cm/h but not nearly 3.3 cm/h. So again the coefficient appears to be in the same general range of the hydraulic and wind values.

For the sake of completeness of the case study the contribution to reaeration by the dam upstream at Andresy needs to be included. Reaeration estimates across dams are normally quantified as the fraction dissolved oxygen increase over the catarat. The early work of Gameson, and later others, used this approach; these dam correlations have been complied by McCutcheon [7]. The fraction DO is computed by a dimensionless empirical parameter r_d which includes such variables as water quality, water level difference, weir type, temperature, etc. Recasting the algorithm in terms of a reaeration coefficients yields

$$k_2' = Q(1 - 1/r_d)/Wl \tag{16}$$

where Q is the discharge in cm^3/h and l is the distance between adjacent dams in cm. Over the river section near Porcheville and downstream, for the 2.8 m dam at Andresy $Q = 736\,m^3/s$, $W = 1$ km, $l = 49$ km and $r = 1.17$ [1] this yields $k_2' = 0.8$ cm/h. This, therefore, is the equivalent reaeration coefficient contributed by the dam at Andresy. Aeration due to the presence of these catarats is accounted for in QUAL 2-type models for the Seine, of which thirteen exist between Montereau and Poses.

Using the KALITO model, Even and Poulin [1] have determined that $k_2 = 0.28$/d does a reasonable job of characterizing the reaeration of the river that includes the Porcheville stretch. With the depth of 5 m this yields $k_2' = 5.83$ cm/h. The contribution to aeration by dams on this stretch is *not* included in this k_2' value. Fig. 7 shows the KALITO value along with the coefficient values for the three mechanisms of hydraulic, wind and vessels. Due to the uncertainties mentioned, the authors are not attaching any significance to the near coincidence of the sum of flow, wind and vessel coefficients with that of the KALITO model. What is encouraging is that the range of the summed model values roughly approximate a k_2' value derived from an independent data set. The KALITO value was a 'best guess' choice of k_2' that meshed well with the existing oxygen data on the river.

There are many assumptions in the theoretical model and several arbitrary, numerical values were chosen for the parameters in order to perform the motor vessel aeration calculation. Although the momentum and aeration concepts used in the model development were based on well known principles, alternative algorithms could be used for the vessel drag characterization, for reaeration and for river hydraulics. It is doubtful that other choices for these would have resulted in grossly different (i.e., order-of-magnitude) numerical results. Numerically the lowering of R, the propeller rated oxygen delivery from 3.0 to 1.2 g/W h, yields a somewhat better correspondence to the KALITO model result. Obviously this adjustment can be performed with other parameters and it extends to the uncertainties, both larger or smaller, in the parameters used for flow and wind aeration. Nevertheless reasonable and creditable values for all parameters were chosen from the range that was available and in this one case it appears that motor vessel activity explains a significant portion of the atmospheric oxygen supply in the Seine river.

6. Summary and conclusions

On the subject of reaeration formulas McCutcheon [7] observed that "...several factors are not included and that a lack of full appreciation of what is not known about reaeration probably contributes to the order-of-magnitude prediction

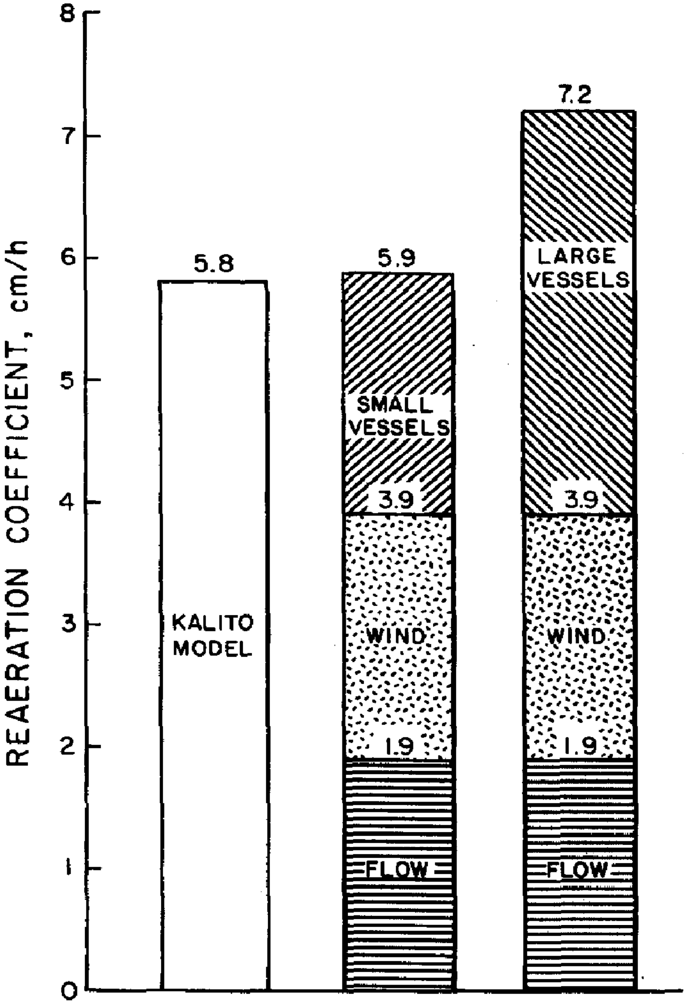

Fig. 7. Decomposition of an overall reaeration coefficient.

errors expected from most k_2 equations". Vessel aeration is one factor that is being excluded!

The primary objective of this paper was to develop a practical algorithm for estimating the contribution to stream aeration provided by motor vessels. In order to illustrate the calculation procedure and compare numerical values with other well-known aeration processes a case study of the Seine river was performed. Based on independent work, a value of the overall reaeration coefficient near Porcheville was available. Using environmental data for the Porcheville section along with available algorithms, the contributions by flow, wind and vessels to the overall coefficient were estimated. It was found that each mechanism contributed approximate equal amounts. Due to the availability of only a single 'observed' value of the overall coefficient detailed perturbation of parameters and statistical analysis of significance were not performed and therefore the case study is essentially an illustration.

However, it does point out, in a numerical sense, the potential importance of vessels in stream aeration and reinforces ones intuition concerning the significance of the process. This significance can be obtained by standing on the bank of streams such as the Seine and experiencing, both visually and audibly, the effects of the vessel generated turbulence on and in the water.

Note added

The authors have been unable to locate other works, either theoretical or experimental, including field observations, on the subject. The mechanism of vessel aeration appears to be in a very crude state of development from both the scientific understanding and engineering applications point-of-view. The model offered in this paper is only a beginning. Others need to be proposed and experimental data, generated in laboratory flow channels fitted with scale model vessels, needs to be obtained. The models can then be tested against this data followed by field tests with real vessels on actual streams to verify the algorithms.

The use of waterbodies for multiple purposes will increase in intensity and the contribution of this and other aeration processes will need to be predicted quantitatively with a high degree of confidence. Unlike the wind or the natural flow reaeration mechanisms including dams, vessels provide a means of interdiction, potentially healing situations of projected or actual low oxygen levels. The problem of procurement and mobilization of vessels to a location of low DO on a waterway is beyond the scope of this paper, however, the potential for this solution exists provided we know how effective the vessel aeration process really is. There appears to be good reasons for continued work on this subject.

Nomenclature

SI units used on all terms.

b	Oxygen consumption rate by bacteria, kg/m^3 s.
C	O_2 concentration in water, C^*–O_2 solubility, kg/m^3.
D	Molecular diffusivity, cm^2/s
f	Friction factor, dimensionless.
F	Friction force, N.
g	Gravitational acceleration, m/s^2.
h	Stream depth, m.
h_v	Vessel draft, m.
k_2	Reaeration coefficient, s^{-1}.
k'_2	Stream O_2 mass-transfer coefficient, m/s.
l	Distance between dams on stream, m.
l_v	Vessel length, m.

n	Coefficient of stream roughness (Eq. (7)), t/m
n_0	O_2 flux across A/W interface, kg/m^2 s.
N	Number of vessels moving past a stream x-section, s^{-1}.
P	Power input by vessel propeller, W.
Q	Stream flow volumetric rate, m^3/s.
r	Stream hydraulic radius (Eq.(7)), m.
r_d	Dam aeration parameter, dimensionless.
S	Stream effective hydraulic gradient, m/m.
SOD	Sediment oxygen demand, kg/m^3 s.
t	Time, s.
v	Wind speed at 10 m elevation, m/s
V	Steam velocity, m/s.
V_e	Effective, vessel enhanced stream velocity, m/s.
V_v	Vessel velocity through water, m/s.
w_v	Vessel width, m.
W	Stream width, m.
α_0	O_2 production rate by photosynthesis, kg/m^3 s.
β	O_2 consumption by algal respiration, kg/m^3 s.
ρ	Density of water, kg/m^3.
τ_v	Flow-induced bottom shear stress, N/m^2.
ν	Kinematic viscosity of water, m^{2}s

References

[1] S. Even and M. Poulin, Modèle de Simulation de l'Ecosysteme Seine schéma Conceptuel Adopté, LHM/RD/90/74, CIG, Ecole Nationale Superieure des Mines de Paris, Fontainebleau.

[2] L.J. Thibodeaux, Chemodynamics, Environmental Movement of Chemicals in Air, Water and Soil, Wiley, New York, 1979.

[3] Tetra Tech, Rates, Constants and Kinetic Formulations in Surface Water Quality Modeling, Prepared for US Environmental Protection Agency, Athens Environmental Research Laboratory, 1978, Athens, Georgia.

[4] Y. Cohen, Mass transfer across a sheared, wavy air–water interface, International Journal of Heat and Mass Transfer, 26 (1983) 1289–1297.

[5] P.D. Lunney, C.S. Springer and L.J. Thibodeaux, Liquid-phase mass transfer coefficient for surface impoundments, Environmental Progress, 4 (1985) 203–211.

[6] R.B. Bird, W.W. Stewart and E.N. Lightfoot, Transport Phenomena, Wiley, New York, 1960, pp. 194, 203.

[7] S.C. McCutcheon, Water Quality Modeling, Vol. 1, Transport and Surface Exchange in Rivers, CRC Press, Inc., Boca Raton, FL, 1989, pp. 235–378.

[8] R.W. Fox and A.T. McDonald, Introduction to Fluid Mechanics, 4th Ed., Wiley, New York, 1992, pp. 507–508.